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Computationally Efficient yet Accurate Analysis of Composite Plate

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Abstract

Many primarily loaded aircraft structure such as skin panels, have the form factor of plates or shells. The Variational Asymptotic Method (VAM) is used to systematically reduce the dimensionality of such structures by taking advantage of the smallness of their thickness vis-à-vis their other dimensions. VAM systematically reduces the 3D elasticity problem into a linear 1-D through-the-thickness analysis and a geometrically nonlinear 2-D plate analysis. The reduced order 2-D model, which is transformed into Reissner-like model, is used to predict 3-D displacement, 3-D stress and 3-D strain. The significant feature of VAM is that it does not use any ad-hoc assumptions on the displacement and stress fields in order to derive the reduced model. Present paper deals with analysis of a composite plate under bending and shear using VAM. The 3-D results recovered from VAM (1-D+2-D) are compared with that obtained from commercial 3-D finite element analysis. A comparative study is carried out to validate the plate results. Acceptable engineering accuracy of the present approach amidst major savings in computational resources is demonstrated with supporting data for both accuracy and efficiency.

Keywords: Composite plate, VAM, 2-D and 3-D FEM

1. Introduction

The main motivation for an aircraft design is to minimize the weight and maximize the performance. This needs to be supported by cost effectiveness of the structure which ensures its acceptability by the industry. To fulfill above requirements researchers are constantly striving for the adaption of new validated technologies at every stage of design, analysis, fabrication and testing of the structure. Use of light weight composites in the aircrafts have been increased drastically due to its directional and tailor-able properties along with its high specific strength and high specific stiffness. But the anisotropy of these materials makes it difficult to understand their structural behavior without going for extensive design, analysis and testing. This results in huge cost and large time investment. To avoid these researchers trying to use various reduced order models. These reduced order models are based on various kinematic assumptions that can be easily accepted in case of isotropic materials. But for accurate modeling of composite material one need to take into account all possible deformations. So in order to formulate the problem more accurately one must minimize all *ad hoc* kinematic assumptions. Berdichevsky developed the Variational Asymptotic Method to construct a reduced model which is asymptotically correct for dimensionally reducible structures such as plates/shells etc. [1].

The primary load carrying structure of an aircraft consists of plate or shell type of structure. In the preliminary stage of design this reduced order models developed using VAM can be used for accurate prediction of stress strain and displacements. Thus without going for time consuming and costly 3-D finite element analysis accurate prediction of through-the-thickness 3-D field variables have become possible using VAM integrated 2-D nonlinear FEM.

2. Formulation and definition of problems

The present paper deals with analysis of composite plate subjected to cylindrical bending. An asymptotically correct reduced order model developed by Yu et al adopting the Variational Asymptotic Method (VAM) introduced by Berdichevsky [1], is used here. This new approach for modeling composite laminates while taking the advantage of reducibility of the structures is implemented in a computer program named VAPAS developed by Yu *et al* [2][3][4][5]. Here 3-D anisotropic elasticity problem is first fit into a fundamental form while accommodating arbitrarily large displacement and global rotation subject only to the strain being small. Energy functional can be constructed for this nonlinear 3-D problem in terms of 2-D generalized strain measures and warping functions describing the deformation of the transverse normal:

$$\Pi = \Pi(\epsilon_{11}, \epsilon_{12}, \epsilon_{22}, K_{11}, K_{12}, K_{22}, \omega_1, \omega_2, \omega_3) \qquad (1)$$

The $\in_{11}$, $\in_{12}$, $\in_{22}$, K_{11}, K_{12} and K_{22} are the generalized strains and ω_1, ω_2 and ω_3 are the unknown warping functions to be solved later unlike other reduced order theories where it is assumed *a priori*. While taking advantage of smallness of thickness compared to other two dimensions in plate, the VAM is used for the asymptotic expansion of the 3-D energy functional into a series of 2-D energy functional in terms of the small parameters (h/l) such that

$$\Pi = \Pi_0 + \Pi_1\left(\frac{h}{l}\right) + \Pi_2\left(\frac{h^2}{l^2}\right) + o\left(\frac{h^2}{l^2}\right) \qquad (2)$$

where Π_0, Π_1, Π_2 are governing functional for different orders of approximation. After solving for warping functions one can substitute back them into energy functional in equation (1) to obtain 2-D energy functional for 2-D plate analysis. The zeroth order approximation for a plate model is of the form

$$\Pi = \Pi(\epsilon_{11}, \epsilon_{12}, \epsilon_{22}, K_{11}, K_{12}, K_{22}) \qquad (3)$$

The energy functional for the zeroth order approximation Π_0 coincides to the CLT; but it is obtained without invoking the Kirchhoff hypothesis unlike the classical treatment, the transverse normal is flexible during deformation. The limitations of other higher order approximations are minimized by VAM as it uses exact kinematical relations between derivatives of the generalized strains of CLT and the transverse shear strains along with equilibrium equations. VAM also ensures asymptotic correctness through second order by imposing minimization techniques while finding out the transverse shear energy. Thus, the loss of accuracy between the asymptotically correct model and a generalized Reissner-Mindlin model is minimized mathematically. Yu implemented a finite element discretization in the computer program VAPAS for through-the-thickness analysis. In the present work 1-D through-the-thickness analysis is carried out using a computer program namely VAPAS developed by Yu and 2-D nonlinear plate analysis is carried out using ABAQUS®. The generalized strains obtained by ABAQUS® after integrating VAM are then used to calculate 3-D field variables like 3-D displacements, 3-D strains and 3-D stresses. Thus without going for time consuming and costly 3-D finite element analysis one can predict 3-D result using VAM integrated 2-D FEM.

Figure 1 show the plate studied here. It is subjected to out-of-plane pressure load (*p*). The plate size is 360x12x1.35 mm^3 (LxBxH). The boundary conditions considered here are clamped on all the four edges. The lay-up sequence of the plate is: $[-45/-45/45/45]_{s.}$ T900/3900-2 graphite epoxy tape is used here and their properties are as below:
E11 = 160GPa; E22 = E33 =9GPa; G_{12} = G_{13} = 6.2GPa; G_{23} = 3.5GPa

$\nu_{12} = \nu_{13} = 0.28$; $\nu_{23} = 0.36$

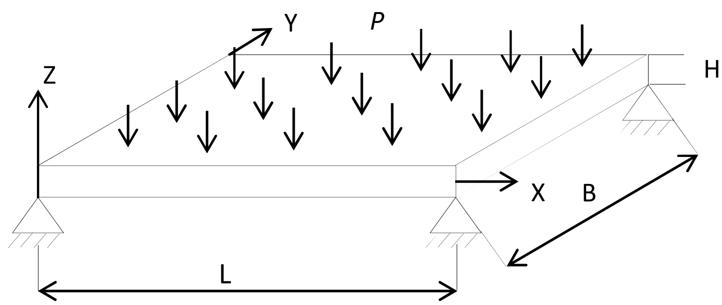

Fig. 1. Composite plate subjected to out-of-plane load

3. Results and Discussion

For the composite plate, 2-D constitutive law is obtained using VAPAS which includes A, B and D matrix and a transverse stiffness matrix G. They are used as an input for the non-linear finite element analysis carried out using ABAQUS® are pre- and post- processor. 2-D generalized strains and displacement are used in VAPAS input file to obtain recovery relations. The recovery relations here mean 3-D displacements, 3-D stress and 3-D stress.

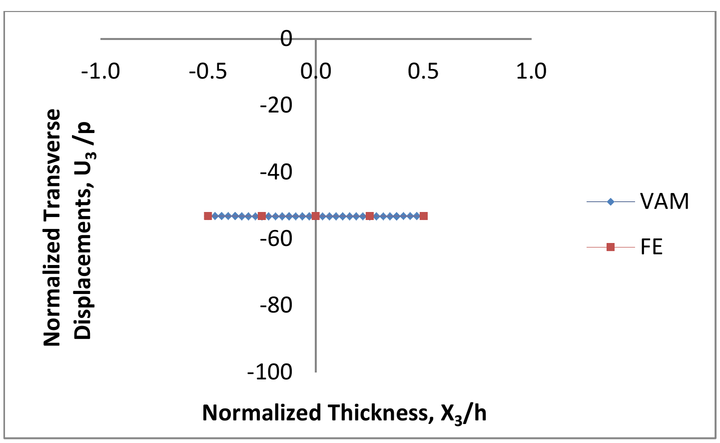

Fig. 2. Distribution of normalized transverse displacement through-the-thickness

For presenting the results graphically the stresses and displacements are normalized as follows:

$$S_{ij} = \frac{\bar{S}_{ij}}{p} \tag{4}$$

$$\bar{Z} = \frac{X_3}{h} \tag{5}$$

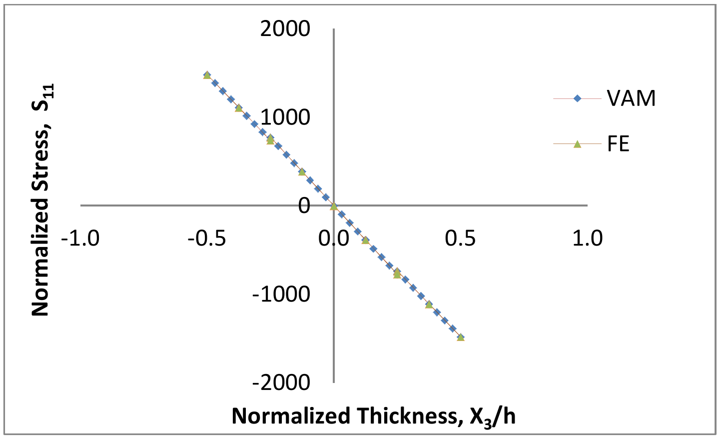

Fig. 3. Distribution of normalized stress S_{11} through-the-thickness

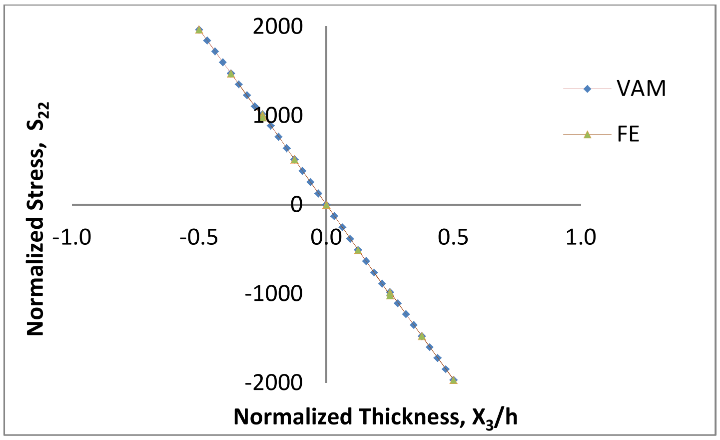

Fig. 4. Distribution of normalized stress S_{22} through-the-thickness

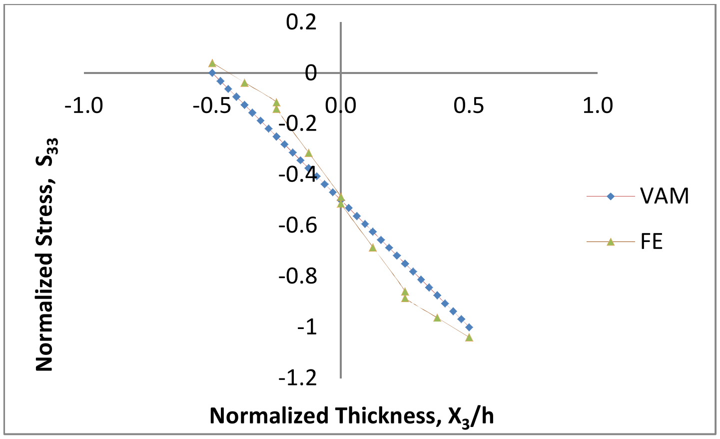

Fig. 5. Distribution of normalized stress S_{33} through-the-thickness

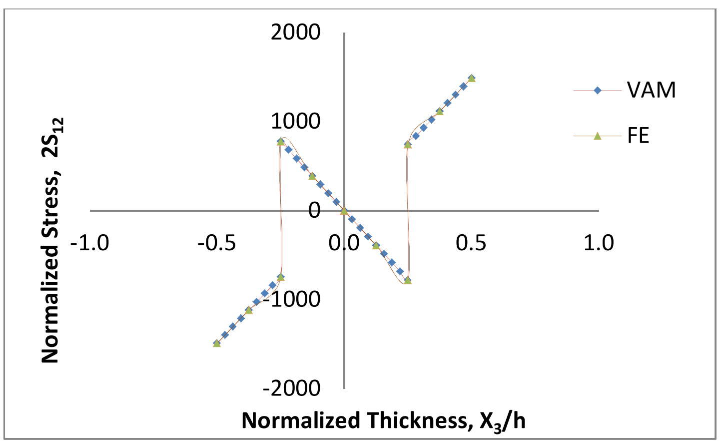

Fig. 6. Distribution of normalized stress $2S_{12}$ through-the-thickness

The results thus obtained by VAM are validated with 3-D finite element analysis results obtained using ABAQUS®. Figure 2 shows the distribution of normalized transverse displacements through-the-thickness. The displacement obtained from VAM and FEM showed a very good agreement.

Figures 3 to 6 show the variation of normalized stress S_{11}, S_{22}, S_{33} and $2S_{12}$ through the normalized thickness (X_3/h). Here S_{11}, S_{22} and S_{33} are the normalized stresses in X, Y and Z direction of the plate and S_{12} is the normalized shear stress in XY plane. From

Normalized stress S_{11}, S_{22} and S_{12} show a very good agreement of VAM results with 3D FEM results.

Table 1 shows the validation of VAM results with 3D FEM results. The % deviation observed in plate results is quite acceptable. Thus a reduced order model is able to predict 3-D stress tensor, 3-D strain tensor and 3-D displacements accurately.

Table 1: Validation of normalized VAM results with normalized 3-D FEM results

Normalized Thickness (Z)	% change in VAM and FE results				% change in VAM and FE results			
	S_{11}	S_{22}	S_{33}	$2S_{12}$	E_{11}	E_{22}	E_{33}	$2E_{12}$
0.5	0.0	0.0	-3.9	0.0	-0.03	0.0	0.03	0.63
0.25	0.24	0.22	-18.1	0.06	0.03	0.15	0.3	-4.11
0	0.7	6.91	2.61	0	0	0	2.7	0
-0.25	0.24	0.22	43.5	0.06	0.03	0.15	0.3	-4.11
-0.5	0.0	0.0	0.0	0.0	-0.03	0.0	0.02	0.63

4. Conclusions

Through-the-thickness analysis of composite plate shows a very good match between VAM and 3-D FEM. The validation of VAM results with 3-D FEM showed variations well within the acceptable limits. The time taken by VAM is very less compared to the time taken by 3-D FEM. So a reduced order model obtained using VAM results in accurate prediction of 3-D field variables like displacements, stress and strain. Without going for extensive 3-D FE modeling acceptable level of accuracy is achieved between a reduced order VAM plate model and 3-D FE model. Thus VAM ensures a cost-effective and accurate analysis of composite plate.

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